A MATHEMATICAL MODEL FOR FREUD'S "PROJECT"

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RESUMO

Apresentamos um modelo matemático para a metapsicologia proposta por Freud em seu trabalho conhecido como o **Projeto para uma Psicologia Científica.** *O modelo é conseqüência de uma associação entre os três arquetipos fundamentais da Física-Matemática e os três sistemas de neurônios propostos por Freud.*

Palavras-chaves: *energia neural, princípio da inércia neural, arquétipos parabólico, elíptico e hiperbólico.*

ABSTRACT

We present a mathematical model for Freud's proposed methapsychology in his work known as Project for a Scientific Psychology. The model is consequence of association among the three fundamental archetypes of Physical Mathematics and three neuron systems proposed by Freud.

Keywords: *neural energy, neural inertia principle, neural theory, parabolic, elliptic and hyperbolic archetypes.*

1. INTRODUCTION

During 1895, Freud formulated a theory with neurophysiological profile, for human mind functioning. It appeared in 1950, after Freud's death. The manuscript named by James Strachey, organizer of the Standard Edition of Complete Works of Sigmund Freud, was "Project for a Scientific Psychology" ,although Freud had referred to it as "Project for Neurologists", in a letter (32nd letter) to his friend Wilhelm Fliess.

In 1952, Hodgkin and Huxley proposed a mathematical model to describe ionic and electric phenomena occurring during an electrical impulse transmission along an axon of a giant squid. This model has a black box which is exactly the membrane of the axon and it is the most important mathematical model in neurology [3]. Although both Freud's Project and Hodgkin-Huxley's model have in common Helmholtz's works and, according to opinion of Karl Pribram (a neuroscientist), Freud having tried to find the equations of the field produced by nervous system, the works have radically distinct focus, observing that the last one is restricted irremediably to a particular neurophysiology phenomenon, while the Project, according to Pribram [2]: "... it is, in a sense, a Rosseta stone for those interested in getting possible communication between neurological and behavioral domains of speech".

The existing gap between the psychological and mathematical sciences is already being confronted, in psychology by scientists such as Kurt Lewin with his "Topological Psychology" and W. R. Bion with his "*pseudomathematical*" propositions. It is possible to foresee a way, from the mathematical side, capable of shortening the gap, and this away is deeply related to the Project.

2. THE NEURAL ENERGY CONCEPT

This is doubtless the most important concept of the Project. Its implications and unfolding are fundamental for understanding the brain-mind interface, since neural energy is the "physical support" of human mind itself. Freud defines the "amount of neural energy", denoted by Q, for being responsible for neuron activation, present the following properties: growing, decreasing, moving, retention and discharging. The neuroelectric manifestation of Q while occupying a neuron (or a neuron cluster) is represented by Qη'. Thus, Qη' may be interpreted as a local charge of neural energy that occupies or "*cathects*" (a freudian term) a neuron. Again, quoting Pribram: "Summing up, we believe that Q, quantity, refers to the physical and chemical amount of energy; Qη'is its neuroelectric manifestation, which may stored up as a potential within a neuron, and convert itself in streams of actions of nervous impulse; the streams discharge the neuron whenever they overcome the resistance in the contacts neurons".

3. THE FIRST POSTULATE

The Project was conceived from two basic postulates: the first is the neural inertia principle, developed from concepts such as the neural excitation and discharge amount, specific action, storing and effort; the second is made up by a neural theory yielding the temporal-space structure of the neural net. The inertia principle may be synthesized in the following assertion: the neurons tend to rid of Q. This is the fundamental law which Q (and therefore Qη') must satisfy. On the other hand, the following considerations are found in tne Project: "From the beginning, however, the inertia principle is broken by other circumstance. With an increasing complexity from the inner organism, the nervous system gets stimuli from the somatic element itself—endogenous stimuli—that must be discharged too. Those stimuli are born organism cells and create the great needs: hunger, breathing, sexuality. The organism can not get rid of them, as opposed to what it does with outside stimuli, it can not use their Q to get away from the stimulus. They only ceased under certain conditions, which must come off in the outside environment. For undertaking that action (which deserves to be qualified as "specific") an independent performance is required from the endogenous Qη' and generally it must be large than them, because the individual is subjected to conditions which may be described as *life demands*. Consequently, the nervous system must given up its original tendency to inertia. It must learn to tolerate enough accumulation of Qη' to satisfy requirements of specific action. Even so, the way it accomplishes that shows the same tendency is maintained, modified by attempting to maintain Qη' at least at the lowest possible level and to guard itself against any increase of Qη' — that is, to keep it constant." **[1](p. 296)**

That lowest possible level may be interpreted as being the necessary energy for keeping basal metabolism, and will be named Q_0. Thus we may sum up the neural inertia principle as the following neural energy decay law

Postulate I : ***There is*** $Q_0 > 0$, ***such that***
$Q(t) \rightarrow Q_0$, ***when*** $t \rightarrow +\infty$.

4. THE SECOND POSTULATE

Freud built his neural theory from a neural system (about 10^{12} neurons) division in three functionally distinct systems: Φ, Ψ and Ω. These three systems are characterized by presenting, respectively, the following behaviors when occupied by Q: discharge, charge and temporal periodicity. These behaviors, themselves, are result of the respective proprieties credited by Freud to the neural systems Φ, Ψ and Ω, related to Q: "permeability", "non-permeability" and "periodicity".

With this functional division of the neural system, Freud derived the primary processes and consciousness (respectively associated to the Φ, Ψ and Ω systems). Here a crucial observation is necessary: behaviors presented by Φ, Ψ and Ω systems are actually the behavior of Qη', when it *cathects* or occupies respectively each of the Φ, Ψ and Ω systems. Thus environments may be defined where those "occupations" (interactions) happen through the following

Definition I:
Qη'(Φ) *is the "discharge" universe.*
Qη'(Ψ) *is the "charge" universe.*
Qη'(Ω) *is the "vibration" universe.*

On the other hand, there is in the Project the following characterization (still current, according to Pribram) of the neuronal net: "The nervous system consists of distinct neurons, structurally homogeneous, that keep contact with one another through strange substance..." [1] (p. 298). This strange substance, the interstitial substance, suggests Freud , conceived the three neural systems as disjoint subsets. Besides, one has to consider that the number of synaptic connections as well as the dendrite trees geometry itself changes with time, a fact found out afterwards! [4]. Thus, naming the neural system SN, the SN time-space structure is analog the non-cylindrical domains [5], in fact

$$SN \subset R^3 \times [0,+\infty[$$

where

$$SN = \bigcup_{t\geq 0} SN_t$$

where $SN_t = SN \cap (R^3 x \{t\}),\ t \geq 0$.

Thus, we get the following from Freud's neural theory

Postulate II: $SN = \Phi \cup \Psi \cup \Omega$, *with* $\Phi \cap \Psi \cap \Omega = \emptyset$.

However, taking into account the SN time-space structure, we get the following result

Corollary:

$$SN = \bigcup_{t\geq 0} (\Phi_t \cup \Psi_t \cup \Omega_t),\ where\ \Phi_t \cap \Psi_t \cap \Omega_t = \emptyset, t \geq 0.$$

5. THE MATHEMATICAL MODEL

5.1 The equations

Everyone who got involved with Mathematical-Physics was introduced to three fundamental archetypes: the heat operator, the Laplacian operator and the wave operator or D'Alembertian. These three differential operators described diffusion, potentiality and vibration phenomena respectively. Before this qualitative and quantitative intriguing coincidence, we put the following question: why have matnematical physical sciences arrived exactly at these three archetypes?

We believe the Project presents the following answer to that question: because human being is organically equipped with three neural systems which show exactly the same proprieties described by such archetypes!

Considering then the sets Qη'(Φ), Qη'(Ψ) and Qη'(Ω) as functional spaces where *cathectical* manifestations of Qη' inhabit, we get a first and fundamental correspondence between methapsychology presented in the Project and physical mathematical sciences formulated in the following result

Postulate III:

Qη'(Φ) is the space of solutions of the diffusion archetype.
Qη'(Ψ) is the space of solutions of the potential archetype.
Qη'(Ω) is the space of solutions of the vibration archetype.

Now, we are able to get the equations of the mathematical model for the Project. In fact, for mathematical characterization of Qη'(Φ) we shall use a parabolic partial differential equation, for characterization of Qη'(Ψ) we shall use a elliptic partial differential equation and for characterization of Qη'(Ω) we shall use a hyperbolic partial differential equation.

On the other hand, Freud builds his methapsychology shown at the Project from the interaction among the elements of Qη'(Φ), Qη'(Ψ) and Qη'(Ω). Therefore, it is necessary, at least from the beginning, to consider the possibility of interactions (couplings) among the elements of Qη'(Φ), Qη'(Ψ) and Qη'(Ω) (one might call them "psychic fields"). With that in mind, having those elements generically represented by φ, ψ and ω respectively, we are led in a first approach to the following partial differential equations system:

$$\begin{cases} \varphi_t - \Delta_3\varphi = F(\varphi,\psi,\omega,\nabla_3\varphi,\nabla_3\psi,\nabla_3\omega,\omega_t) \\ \Delta_4\psi = G(\varphi,\psi,\omega,\nabla_4\varphi,\nabla_4\psi,\nabla_t\omega) \\ \omega_{tt} - \Delta_3\omega = H(\varphi,\psi,\omega,\nabla_3\varphi,\nabla_3\psi,\nabla_3\omega,\varphi_t,\omega_t) \end{cases}$$

where $(x,t) \in SN$, ∇_n and Δ_n stand for the n-dimensional gradient and Laplacian respectively. This system shows a structural problem of compatibility, for if the ψ field has a time-independent character how to interpret the time variable in the second equation? A solution would be to take the time variable in the second equation as a parameter, obtaining a family of elliptic equations for the ψ field. However, that would not attend the following demand stated by Freud: "... the consciousness contents shall lie among our quantitative ψ processes" [1] (p. 308). Following that clue, we shall be able to extract the ω processes from the ψ processes, that is, in mathematical terms, we must extract the three-dimensional D'Alembertian operator from the Laplacian operator and vice-versa. One way of accomplishing this breakthrough is to considerate that the elements of Qη'(Ψ) undergo a "*spacialization*" in the temporal variable through the following transformation: $x_4 = it$. With that, one gets the Laplacian operator (four-dimensional), actually the $-\Delta_4$ operator, from the D'Alembertian operator (three-dimensional). And, reciprocally, if one "*temporizes*" space though the mapping: $t = -ix_4$, we shall obtain the

D'Alembertian operator (four-dimensional). Thus, in the second equation the gradient and the Laplacian are four-dimensional, while in the other two equations they are tri-dimensional.

At this point, we remind that Qη'(Φ), Qη'(Ψ) and Qη'(Ω) are functional spaces, that is φ, ψ and ω are functional defined over Φ, Ψ, and Ω respectively. Here a technical problem comes off: dual spaces are necessarily vectorial spaces! However, in the Project we find the following conjecture: "If 1(Qη') in φ yields a cathects in ψ, then 3(Qη') shows itself through a cathect in $\psi_1 + \psi_2 + \psi_3$..." [1](p. 314). Therefore one may find somewhat of a backing considering Qη'(Φ), Qη'(Ψ) and Qη'(Ω) as (topological) vectorial spaces.

In the proposed system we are considering couplings in their most possible generality, but without destroying the archetypal characterization of each equation.

5.2 *Initial conditions*

Differently from Hodgkin-Huxley equations, one has not here enough biochemical data for a precise definition of initial conditions, particularly about neurotransmitters and ion channels in synaptic openings (slits). Moreover, another obstacle is generated by a technical problem: existing mathematical methods only apply to non-cylindrical domains whose projections in R^3 form a non-decreasing sequence of nested subsets! If that is attended, initial conditions will have to be taken in SN_0, being given by

$$\varphi(x,0) = \varphi_0(x), x \in \Phi_0$$
$$\omega(x,0) = \omega_0(x), x \in \Omega_0$$
$$\omega_t(x,0) = \omega_1(x), x \in \Omega_0$$

where $x = (x_1, x_2, x_3) \in R^3$.

Remark: Since one has $\omega(x,t) = \psi(\underline{x})$, where $\underline{x} = (x_1, x_2, x_3, it)$, then

$$\omega(x,0) = \psi(x,0) = \psi|\Psi_0$$

that is, taking a initial condition for ω is the same as taking a boundary condition for ψ.

5.3 *Boundary conditions*

The role played by the boundaries of the three neural systems, Φ, Ψ, and Ω in the Project is the fundamental, because those boundaries have the synaptic slits (named by Freud as *"contact barriers"*) which are responsible for passing Qη' from system to the other and where Freud located all the resistance's to Qη' passage. In fact, it was through the *"barriers"* behavior relating to Qη' passage that Freud characterized the three neural systems. However, in order to study the "well-posed" question in Hadamard's sense from Cauchy's problem to the Freud's system, one needs to know the differentiability class of the boundaries which is itself related to SN geometry. It is a delicate question, but it appears the best geometry for modeling SN is the fractal one. That causes a technical problem, for if we decide in favor of fractal geometry, the boundary certainly will present no differentiability, which will call for adaptations of Lopatinski-Shapiro's conditions and the known trace theorems. In a first approach, however, we may, at least formally, take the boundary conditions

$$\alpha_1(x)\varphi(x) + \beta_1(x)\frac{\partial\varphi}{\partial\eta}(x) = f(x), x \in \partial\Phi,$$
$$\alpha_2(\underline{x})\psi(\underline{x}) + \beta_2(\underline{x})\frac{\partial\psi}{\partial\eta}(\underline{x}) = g(\underline{x}), \underline{x} \in \partial\Psi$$
$$\alpha_3(x)\omega(x) + \beta_3(x)\frac{\partial\omega}{\partial\eta}(x) = h(x), x \in \partial\Omega,$$

$t > 0$, where because the permeability property one have $\beta_i \neq 0$, $x \in \partial\Omega_i$, $t > 0$ and because the non-permeability one have $\beta_2 = 0$, almost always in ∂Ψ. On the other hand, while approaching the memory's characterization problem the Project tells us that: "Memory is represented by existing facilities (selective) among the ψ neurons." [1] (p. 299). Therefore one might also consider boundary conditions given by oblique derivatives applied to ∂Ψ subsets, of positive value, and whose direction field would characterize the way of facilities.

5.4 *Conclusions*

As one observes, the model was built in the most general form, for coupling among fields were the most generic ones, using the most simple form, using canonical representatives of the parabolic, elliptic and hyperbolic archetypes. In a first approach, one might consider Freud's system as also presenting the most simple couplings, but still representing an effective interaction among the three fields, that is, the system

$$\begin{cases} \varphi_t - \Delta_3\varphi = F(\varphi,\psi,\omega) \\ \Delta_4\psi = G(\varphi,\psi,\omega) \\ \omega_{tt} - \Delta_3\omega = H(\varphi,\psi,\omega) \end{cases}$$

where $(x,t) \in SN$. Also, in the Project Freud characterized and related certain types of thinking (defined as *specific secondary processes*) which might get a particular systemic characterization. For example, an endogenous conscious mental process could be modeled by following system

$$\begin{cases} \Delta_4\psi = G(\psi,\omega) \\ \omega_{tt} - \Delta_3\omega = H(\psi,\omega) \end{cases}$$

where $(x,t) \in \Psi \cap \Omega$. Thus, one may perceive the possibility of a systemic characterization in the theory of mental processes, as proposed by Freud.

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