

$$\begin{aligned}\lim_{x \rightarrow +\infty} \frac{x^2 + 3x - 1}{x^3 - 2} &= \lim_{x \rightarrow +\infty} \frac{\frac{1}{x} + \frac{3}{x^2} - \frac{1}{x^3}}{1 - \frac{2}{x^3}} \\ &= \frac{\lim_{x \rightarrow +\infty} \left(\frac{1}{x} + \frac{3}{x^2} - \frac{1}{x^3} \right)}{\lim_{x \rightarrow +\infty} \left(1 - \frac{2}{x^3} \right)} \\ &= \frac{0}{1} \\ &= 0.\end{aligned}$$

(x) Mostrar que se $P(x) = a_0x^n + a_1x^{n-1} + \dots + a_n$ e $Q(x) = b_0x^m + b_1x^{m-1} + \dots + b_m$, então

$$\lim_{x \rightarrow \pm\infty} \frac{P(x)}{Q(x)} = \lim_{x \rightarrow \pm\infty} \frac{a_0x^n}{b_0x^m}.$$

Temos,

$$\begin{aligned}\lim_{x \rightarrow \pm\infty} \frac{P(x)}{Q(x)} &= \lim_{x \rightarrow \pm\infty} \frac{a_0x^n + a_1x^{n-1} + \dots + a_n}{b_0x^m + b_1x^{m-1} + \dots + b_m} \\ &= \lim_{x \rightarrow \pm\infty} \frac{x^n \left(a_0 + \frac{a_1}{x} + \dots + \frac{a_{n-1}}{x^{n-1}} + \frac{a_n}{x^n} \right)}{x^m \left(b_0 + \frac{b_1}{x} + \dots + \frac{b_{m-1}}{x^{m-1}} + \frac{b_m}{x^m} \right)} \\ &= \lim_{x \rightarrow \pm\infty} \frac{x^n}{x^m} \cdot \lim_{x \rightarrow \pm\infty} \frac{\left(a_0 + \frac{a_1}{x} + \dots + \frac{a_{n-1}}{x^{n-1}} + \frac{a_n}{x^n} \right)}{\left(b_0 + \frac{b_1}{x} + \dots + \frac{b_{m-1}}{x^{m-1}} + \frac{b_m}{x^m} \right)} \\ &= \lim_{x \rightarrow \pm\infty} \frac{x^n}{x^m} \cdot \frac{a_0}{b_0} \\ &= \lim_{x \rightarrow \pm\infty} \frac{a_0x^n}{b_0x^m}.\end{aligned}$$

3.13 Exercícios

1. Se $f(x) = \frac{3x + |x|}{7x - 5|x|}$, calcule:

(a) $\lim_{x \rightarrow +\infty} f(x)$.

(b) $\lim_{x \rightarrow -\infty} f(x)$.

2. Se $f(x) = \frac{1}{(x+2)^2}$, calcule:

(a) $\lim_{x \rightarrow -2} f(x)$.

(b) $\lim_{x \rightarrow +\infty} f(x)$.

Nos exercícios 3 a 40 calcule os limites.

3. $\lim_{x \rightarrow +\infty} (3x^3 + 4x^2 - 1).$
4. $\lim_{x \rightarrow +\infty} \left(2 - \frac{1}{x} + \frac{4}{x^2} \right).$
5. $\lim_{t \rightarrow +\infty} \frac{t+1}{t^2+1}.$
6. $\lim_{t \rightarrow -\infty} \frac{t+1}{t^2+1}.$
7. $\lim_{t \rightarrow +\infty} \frac{t^2 - 2t + 3}{2t^2 + 5t - 3}.$
8. $\lim_{x \rightarrow +\infty} \frac{2x^5 - 3x^3 + 2}{-x^2 + 7}.$
9. $\lim_{x \rightarrow -\infty} \frac{3x^5 - x^2 + 7}{2 - x^2}.$
10. $\lim_{x \rightarrow -\infty} \frac{-5x^3 + 2}{7x^3 + 3}.$
11. $\lim_{x \rightarrow +\infty} \frac{x^2 + 3x + 1}{x}.$
12. $\lim_{x \rightarrow +\infty} \frac{x\sqrt{x} + 3x - 10}{x^3}.$
13. $\lim_{t \rightarrow +\infty} \frac{t^2 - 1}{t - 4}.$
14. $\lim_{x \rightarrow +\infty} \frac{x(2x - 7\cos x)}{3x^2 - 5\text{sen } x + 1}.$
15. $\lim_{v \rightarrow +\infty} \frac{v\sqrt{v} - 1}{3v - 1}.$
16. $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + 1}}{x + 1}.$
17. $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 1}}{x + 1}.$
18. $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 1} - \sqrt{x^2 - 1}).$
19. $\lim_{x \rightarrow +\infty} x(\sqrt{x^2 - 1} - x).$
20. $\lim_{x \rightarrow +\infty} (\sqrt{3x^2 + 2x + 1} - \sqrt{2x}).$
21. $\lim_{x \rightarrow +\infty} \frac{10x^2 - 3x + 4}{3x^2 - 1}.$
22. $\lim_{x \rightarrow -\infty} \frac{x^3 - 2x + 1}{x^2 - 1}.$
23. $\lim_{x \rightarrow -\infty} \frac{5x^3 - x^2 + x - 1}{x^4 + x^3 - x + 1}.$
24. $\lim_{s \rightarrow +\infty} \frac{8 - s}{\sqrt{s^2 + 7}}.$
25. $\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 - 7}}{x + 3}.$
26. $\lim_{x \rightarrow +\infty} (\sqrt{16x^4 + 15x^3 - 2x + 1} - 2x).$
27. $\lim_{s \rightarrow +\infty} \sqrt[3]{\frac{3s^7 - 4s^5}{2s^7 + 1}}.$
28. $\lim_{x \rightarrow +\infty} \frac{\sqrt{2x^2 - 7}}{x + 3}.$
29. $\lim_{y \rightarrow +\infty} \frac{3 - y}{\sqrt{5 + 4y^2}}.$
30. $\lim_{y \rightarrow -\infty} \frac{3 - y}{\sqrt{5 + 4y^2}}.$

31. $\lim_{x \rightarrow 3^+} \frac{x}{x-3}$

32. $\lim_{x \rightarrow 3^-} \frac{x}{x-3}$

33. $\lim_{x \rightarrow 2^+} \frac{x}{x^2-4}$

34. $\lim_{x \rightarrow 2^-} \frac{x}{x^2-4}$

35. $\lim_{y \rightarrow 6^+} \frac{y+6}{y^2-36}$

36. $\lim_{y \rightarrow 6^-} \frac{y+6}{y^2-36}$

37. $\lim_{x \rightarrow 4^+} \frac{3-x}{x^2-2x-8}$

38. $\lim_{x \rightarrow 4^-} \frac{3-x}{x^2-2x-8}$

39. $\lim_{x \rightarrow 3^-} \frac{1}{|x-3|}$

40. $\lim_{x \rightarrow 3^+} \frac{1}{|x-3|}$

3.14 Assíntotas

Em aplicações práticas, encontramos com muita frequência gráficos que se aproximam de *uma reta* à medida que x cresce ou decresce (ver figuras 3.14 e 3.15).

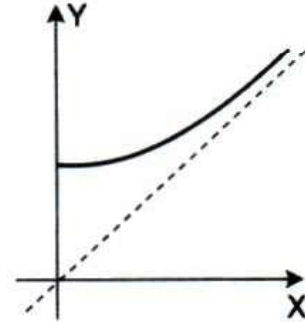
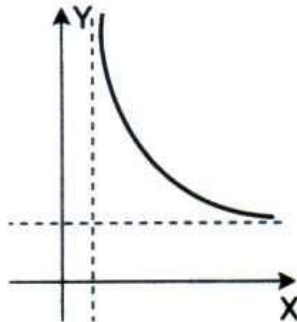
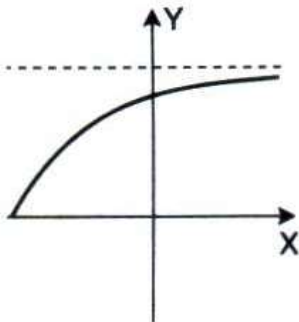


Figura 3.14

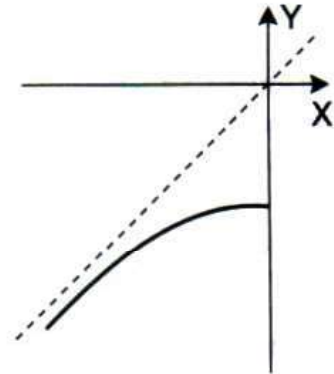
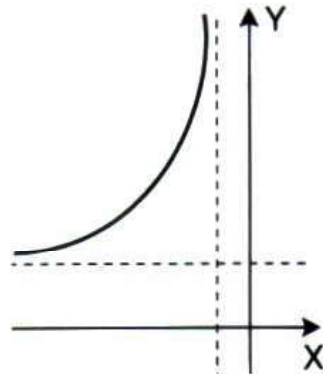
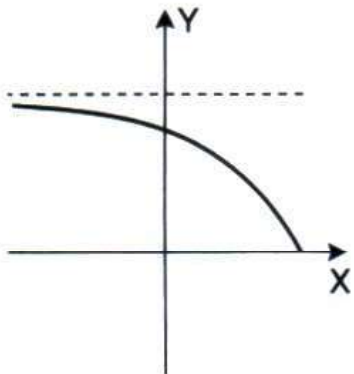


Figura 3.15

Estas retas são chamadas de *assíntotas*.

Particularmente, vamos analisar com um pouco mais de atenção as *assíntotas horizontais* e as *verticais*.