

**UNIVERSIDADE FEDERAL DE OURO PRETO**  
**INSTITUTO DE CIÊNCIAS EXATAS E BIOLÓGICAS**  
**DEPARTAMENTO DE MATEMÁTICA**

Décima Segunda Lista de Exercícios de Cálculo Diferencial e Integral I - MTM122

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21 de Março de 2017

(1) Calcule.

- |   |   |   |   |
|---|---|---|---|
| (1) $\int_0^4 \frac{1}{x^2 + 16} dx$                            | (2) $\int_0^{\sqrt{2}/2} \frac{x}{\sqrt{1-x^4}} dx$                     | (3) $\int \frac{\sin x}{\cos^2(x) + 1} dx$                              | (4) $\int \frac{1}{\sqrt{x}(1+x)} dx$                           |
| (5) $\int \frac{e^x}{\sqrt{16-e^{2x}}} dx$                      | (6) $\int \frac{1}{x(\sqrt{x^6-4})} dx$                                 | (7) $\int \frac{x}{x^2+9} dx$   | (8) $\int \frac{1}{\sqrt{e^{2x}-25}} dx$                        |
| (9) $\int \frac{\sinh \sqrt{x}}{\sqrt{x}} dx$                   | (10) $\int \operatorname{cotgh}(x) dx$                                  | (11) $\int \sinh x \cosh x dx$  | (12) $\int \operatorname{tgh} 3x \operatorname{sech} 3x dx$     |
| (13) $\int \operatorname{tgh}^2 3x \operatorname{sech}^2 3x dx$ | (14) $\int \frac{\operatorname{sech}^2 x}{1-2\operatorname{tgh} x} dx$  | (15) $\int dx$  | (16) $\int \cos(5-3x) dx$                                       |
| (17) $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx$                 | (18) $\int (\operatorname{cotg} 9x + \sec 9x) dx$                       | (19) $\int e^x \operatorname{tg}(e^x) dx$                               | (20) $\int (\operatorname{cosec}(3x) + 1)^2 dx$                 |
| (21) $\int \frac{\sin 4x}{\operatorname{tg} 4x} dx$             | (22) $\int \frac{x}{4+9x^2} dx$   | (23) $\int \frac{e^{2x}}{\sqrt{1-e^{2x}}} dx$                           | (24) $\int \frac{x}{\operatorname{sech}(x^2)} dx$               |
| (25) $\int_{-1/2}^{1/2} \frac{1}{\sqrt{1-x^2}} dx$              | (26) $\int \sec^2 x (1+\operatorname{tg} x)^2 dx$                       | (27) $\int \frac{\operatorname{cosec}^2 x}{2+\operatorname{cotg} x} dx$ | (28) $\int \frac{\sinh(\ln x)}{x} dx$                           |
| (29) $\int \frac{1}{\sqrt{9-4x^2}} dx$                          | (30) $\int \frac{1}{x(\sqrt{9-4x^2})} dx$                               | (31) $\int \frac{x}{\sqrt{25x^2+36}} dx$                                | (32) $\int x(e^{-x}) dx$  |
| (33) $\int x^2(e^{3x}) dx$                                      | (34) $\int x(\cos(5x)) dx$  | (35) $\int x(\sec x \operatorname{tg} x) dx$                            | (36) $\int x^2(\cos x) dx$                                      |
| (37) $\int \operatorname{arctg}(x) dx$                          | (38) $\int \ln x dx$  | (39) $\int \sqrt{x} \ln x dx$   | (40) $\int x(\operatorname{cosec}^2 x) dx$                      |
| (41) $\int e^{-x} \sin x dx$                                    | (42) $\int \sin x \ln(\cos x) dx$                                       | (43) $\int \sec^3 x dx$   | (44) $\int_0^1 \frac{x^3}{\sqrt{x^2+1}} dx$                     |
| (45) $\int_0^{\pi/2} x \sin(2x) dx$                             | (46) $\int x(2x+3)^9 dx$  | (47) $\int e^{4x}(\sin 5x) dx$  | (48) $\int (\ln x)^2 dx$  |
| (49) $\int x^3 \sinh x dx$                                      | (50) $\int \cos(\sqrt{x}) dx$   | (51) $\int \arccos(x) dx$   | (52) $\int \cos^3 x dx$   |
| (53) $\int \operatorname{sen}^2 x \operatorname{cosec}^2 x dx$  | (54) $\int \operatorname{sen}^3 x \operatorname{cosec}^2 x dx$          | (55) $\int \operatorname{sen}^6 x dx$                                   | (56) $\int \operatorname{tg}^3 x \operatorname{sec}^4 x dx$     |
| (57) $\int \operatorname{tg}^3 x \operatorname{sec}^3 x dx$     | (58) $\int \operatorname{tg}^6 x dx$                                    | (59) $\int \sqrt{\sin x} \operatorname{cosec}^3 x dx$                   | (60) $\int (\operatorname{tg} x + \operatorname{cotg} x)^2 dx$  |
| (61) $\int_0^{\pi/4} \operatorname{sen}^3 x dx$                 | (62) $\int \sin 5x \sin 3x dx$  | (63) $\int_0^{\pi/2} \sin 3x \cos 2x dx$                                | (64) $\int \operatorname{cosec}^4 x \operatorname{cotg}^4 x dx$ |
| (65) $\int \frac{\operatorname{cosec} x}{2-\sin x} dx$          | (66) $\int \frac{\operatorname{sec}^2 x}{(1+\operatorname{tg} x)^2} dx$ | (67) $\int \cos\left(\frac{2\pi x}{L}\right) dx$                        | (68) $\int a^x dx$  |

(2) Calcule.

$$\begin{aligned}
 (1) \int \frac{x^2}{\sqrt{4-x^2}} dx & \quad (2) \int \frac{1}{x\sqrt{9+x^2}} dx & (3) \int \frac{1}{x^2\sqrt{x^2-25}} dx & (4) \int \frac{x}{\sqrt{4-x^2}} dx \\
 (5) \int \frac{1}{(x^2-1)^{3/2}} dx & (6) \int \frac{1}{(36+x^2)^2} dx & (7) \int \sqrt{9-4x^2} dx & (8) \int \frac{x}{(16-x^2)^2} dx \\
 (9) \int \frac{x^3}{\sqrt{9x^2+49}} dx & (10) \int \frac{1}{x^4\sqrt{x^2-3}} dx & (11) \int \frac{(4+x^2)^2}{x^3} dx & (12) \int \frac{1}{1+x^2} dx
 \end{aligned}$$

(3) Calcule por substituição.

$$\begin{aligned}
 (1) \int x\sqrt[3]{x+9} dx & \quad (2) \int_4^9 \frac{1}{\sqrt{x+4}} dx & (3) \int_0^4 \frac{2x+3}{\sqrt{1+2x}} dx & (4) \int \frac{e^{2x}}{e^x+4} dx & (5) \int \frac{x^2}{(3x+4)^{10}} dx \\
 (6) \int \frac{\sec x}{4-3\tg x} dx & (7) \int \frac{\sec x}{2\tg x + \sec x} dx & (8) \int \frac{1}{1-\sen x} dx & (9) \int_0^{\pi/2} \frac{1}{3+\cos x} dx & (10) \int \frac{1+\sen x}{1+\cos x} dx
 \end{aligned}$$

(4) Prove.

$$(1) \int \frac{\sqrt{x^2-a^2}}{x^4} dx = \frac{\sqrt{(x^2-a^2)^3}}{3a^2x^3} + c$$

$$(2) \int \frac{1}{x^2(\sqrt{x^2-a^2})} dx = \frac{\sqrt{x^2-a^2}}{a^2x} + c$$

$$(3) \int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$$

$$(4) \int \cos^3 x dx = \frac{1}{3}(2 + \cos^2 x) \sin x + c$$

(5) Seja  $f(x) \geq 0$  em  $[a, b]$ . Se o volume de uma figura gerada pela revolução de um conjunto

$$A = \{(x, y) \in \mathbb{R}^2 / x \in [a, b], 0 \leq y \leq f(x)\}$$

em torno do eixo- $x$  é dado pela integral

$$V = \int_a^b \pi [f(x)]^2 dx,$$

calcule o volume do sólido de revolução nos casos em que

(a)  $f(x) = \sqrt{a^2 - x^2}$  em  $[-a, a]$ ,  $a > 0$ ;

(b)  $f(x) = (a/b)x$  em  $[0, b]$ ,  $a, b > 0$ ;

(c)  $f(x) = a$  em  $[0, b]$ ,  $a, b > 0$ .

(d)  $f(x) = 1/x$  em  $[1, b]$ ,  $b > 1$ .

Desenhe as regiões e os sólidos gerados.