UNIVERSIDADE FEDERAL DE OURO PRETO

INSTITUTO DE CIÊNCIAS EXATAS E BIOLÓGICAS

DEPARTAMENTO DE MATEMÁTICA

Quinta Lista de Exercícios de Cálculo Diferencial e Integral I - MTM122

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(1) Determine se a função $f(x) = -4x^2 - 20x - 25$ tem ponto de máximo ou de mínimo e calcule suas coordenadas. Esboce o gráfico desta função e faça o estudo de sinal.

(2) Construa os gráficos das funções a seguir.

(a) $f(x) = x^2 - 4|x| + 3$ (b) $f(x) = ||x^2 - 1| - 3|$

(c) $f(x) = \left|x^2 - 4|x| + 3\right|$ (d) $f(x) = |\text{sen}2x|$

(e) $f(x) = 2\cos x$ (f) $f(x) = \cos\dfrac{x}{2}$

(g) $f(x) = \text{sen}(\pi x)$ (h) $f(x) = x\text{sen}x$

(i) $f(x) = (1/x)\text{sen}x$ (j) $f(x) = x\text{sen}\dfrac{1}{x}$

(k) $f(x) = x^2\text{sen}\dfrac{1}{x}$ (l) $f(x) = x + \text{sen}(x)$

(m) $f(x) = \text{arcsec}(x)$ (n) $f(x) = \ln(x + \sqrt{x^2 - 1})$

(o) $f(x) = \text{argcosh}(x)$ (p) $f(x) = \dfrac{x^2 - 1}{x - 1}$

(3) Prove as identidades abaixo.

(a) $\text{sen}a\cos b = \dfrac{1}{2}[\text{sen}(a+b) + \text{sen}(a-b)]$ (b) $\cos a\cos b = \dfrac{1}{2}[\cos(a+b) + \cos(a-b)]$

(c) $\text{sen}a\text{sen}b = \dfrac{1}{2}[\cos(a-b) - \cos(a+b)]$ (d) $\cosh^2 t - \text{senh}^2 t = 1$

(e) $\text{argcosh}(x) = \ln(x + \sqrt{x^2 - 1})$, $x \geq 1$ (f) $\text{tg}2\theta = \dfrac{2\text{tg}\theta}{1 - \text{tg}^2\theta}$

(4) Use as propriedades para calcular os limites

(a) $\lim\limits_{x\to 0}(3-7x-5x^2)$

(b) $\lim\limits_{x\to 3}(3x^2-7x+2)$

(c) $\lim\limits_{x\to -1}(-x^5+6x^4+2)$

(d) $\lim\limits_{x\to \frac{1}{2}}(2x+7)$

(e) $\lim\limits_{x\to -1}[(x+4)^3(x+2)^{-1}]$

(f) $\lim\limits_{x\to 1}\dfrac{x^2-1}{x-1}$

(g) $\lim\limits_{t\to 2}\dfrac{t^2+5t+6}{t+2}$

(h) $\lim\limits_{x\to 2}\dfrac{t^2-5t+6}{t-2}$

(i) $\lim\limits_{x\to \sqrt{2}}\dfrac{2x^2-x}{3x}$

(j) $\lim\limits_{x\to 2}\dfrac{x\sqrt{x}-\sqrt{2}}{3x-4}$

(k) $\lim\limits_{x\to \frac{\pi}{2}}[2\text{sen}x-\cos x+\text{cotg}x]$

(l) $\lim\limits_{x\to -1/3}(2x+3)^{1/4}$

[R.3; 8; 9; 8; 27; 2; 5; -1; $\frac{2\sqrt{2}-1}{3}$; $\frac{\sqrt{2}}{2}$; 2; $\sqrt[4]{7/3}$]

(5) Nos items a seguir, verifique as identidades

(a) $\cosh x+\text{senh}x=e^x$

(b) $\cosh x-\text{senh}x=e^{-x}$

(c) $\text{senh}(-x)=-\text{senh}(x)$

(d) $\text{senh}(x+y)=\text{senh}(x)\cosh(y)+\text{senh}(y)\cosh(x)$

(e) $\cosh(-x)=\cosh(x)$

(f) $\cosh(x+y)=\cosh(x)\cosh(y)+\text{senh}(y)\text{senh}(x)$

(g) $\text{senh}(2x)=2\text{senh}(x)\cosh(x)$

(h) $\cosh(2x)=\cosh^2(x)+\text{senh}^2(x)$

(i) $\cosh(x)=\sqrt{\dfrac{1+\cosh(2x)}{2}}$

(j) $[\cosh(x)+\text{senh}(x)]^n=\cosh(nx)+\text{senh}(nx)$, para n inteiro positivo. [Dica:use (a)]

(k) $\text{sen}(x)=\dfrac{2\text{tg}\frac{x}{2}}{1+\text{tg}^2\frac{x}{2}}$

(l) $\cos(x)=\dfrac{1-\text{tg}^2\frac{x}{2}}{1+\text{tg}^2\frac{x}{2}}$

(6) Escreva uma lista contendo as principais propriedades do logaritmo e da exponencial e uma outra contendo as principais identidades trigonométricas.

(7) Calcule os limites a seguir

(a) $\displaystyle\lim_{x\to -1} \frac{x^3+1}{x^2-1}$ (b) $\displaystyle\lim_{x\to 2} \frac{x^2+3x-10}{3x^2-5x-2}$

(c) $\displaystyle\lim_{t\to 0} \frac{(4-t)^2-16}{t}$ (d) $\displaystyle\lim_{h\to 1} \frac{\sqrt{h}-1}{h-1}$

(e) $\displaystyle\lim_{h\to 0} \frac{\sqrt[3]{8+h}-2}{h}$ (f) $\displaystyle\lim_{x\to 3} \frac{x^2-9}{x^2+9}$

(g) $\displaystyle\lim_{x\to 1} \frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}$ (h) $\displaystyle\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

(i) $\displaystyle\lim_{x\to 1} \sqrt[5]{\frac{(3-x^3)^4-16}{1-x^3}}$ (j) $\displaystyle\lim_{x\to -1} \frac{x+1}{\sqrt[3]{x+2}-1}$

(k) $\displaystyle\lim_{x\to \frac{1}{2}} -\frac{\frac{1}{x}-2}{x-\frac{1}{2}}$ (l) $\displaystyle\lim_{h\to 0} \frac{(x+h)^3-x^3}{h}$

(k) $\displaystyle\lim_{x\to a} \frac{\sqrt[3]{x}-\sqrt[3]{a}}{x-a}$, $a\neq 0$ (l) $\displaystyle\lim_{h\to 0} \frac{\sqrt[4]{x}-\sqrt[4]{a}}{x-a}$, $a\neq 0$.

[R. $-3/2; 1; 8; 1/2; 1/12; 0; 1/9; 1; 2; 3; 4; 3x^2; 1/(3\sqrt[3]{a^2}); 1/(4\sqrt[4]{a^3})$ - ver mais exercícios Diva p.103 5ª ed.]

(8) Calcule, se existirem, os limites a seguir

(a) $\displaystyle\lim_{x\to 3} \frac{5x+11}{\sqrt{x+1}}$ (b) $\displaystyle\lim_{x\to 5} \frac{(1/x)-(1/5)}{x-5}$

(c) $\displaystyle\lim_{t\to 0} \frac{t^2}{\sin t}$ (d) $\displaystyle\lim_{x\to 0} \frac{x^2+\sin^2 x}{4x^2}$

(e) $\displaystyle\lim_{x\to 0} \frac{\sin^2 x+\sin(2x)}{3x}$ (f) $\displaystyle\lim_{x\to 0} \frac{2-\cos x}{1+\sin x}$

(g) $\displaystyle\lim_{x\to 0} \frac{2\cos x+3x-2}{5x}$ (h) $\displaystyle\lim_{x\to 0} \frac{3x+1-\cos^2 x}{\sin x}$

(i) $\displaystyle\lim_{x\to 0} \frac{x\sin x}{1-\cos x}$ (j) $\displaystyle\lim_{x\to 0} \frac{\cos(x)-1}{2x}$ [$R.13; 0; 2/3; 3/5; 2.$]

(9) Sejam f e g duas funções com mesmo domínio A tais que

$$\lim_{x\to x_0} f(x) = 0$$

e g é uma função limitada (ou seja, existe $M>0$ tal que $|g(x)|\leq M$ para todo $x\in A$). Use o Teorema do Confronto (Sanduíche) para provar que

$$\lim_{x\to x_0} f(x)g(x) = 0.$$

[R. São contínuas, b,c,d.]

(10) Calcule.

(a) $\displaystyle\lim_{x\to+\infty}(3x^3+4x^2-1)$

(b) $\displaystyle\lim_{t\to+\infty}\frac{t+1}{t^2+1}$

(c) $\displaystyle\lim_{t\to+\infty}\frac{t^2-2t+3}{2t^2+5t-3}$

(d) $\displaystyle\lim_{x\to-\infty}\frac{3x^5-x^2+7}{2-x^2}$

(e) $\displaystyle\lim_{x\to+\infty}\frac{x^2+3x+1}{x}$

(f) $\displaystyle\lim_{x\to+\infty}\frac{x^2-1}{x-4}$

(g) $\displaystyle\lim_{\nu\to+\infty}\frac{\nu\sqrt{\nu}-1}{3\nu-1}$

(h) $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{x^2+1}}{x+1}$

(i) $\displaystyle\lim_{x\to+\infty}x(\sqrt{x^2-1}-x)$

(j) $\displaystyle\lim_{x\to+\infty}\frac{10x^2-3x+4}{3x^2-1}$

(k) $\displaystyle\lim_{x\to-\infty}\frac{5x^3-x^2+x-1}{x^4+x^3-x+1}$

(l) $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{2x^2-7}}{x+3}$

(m) $\displaystyle\lim_{s\to+\infty}\sqrt[3]{\frac{3s^7-4s^5}{2s^7+1}}$

(n) $\displaystyle\lim_{y\to+\infty}\frac{3-y}{\sqrt{5+4y^2}}$

(o) $\displaystyle\lim_{x\to3^+}\frac{x}{x-3}$

(p) $\displaystyle\lim_{x\to3^-}\frac{x}{x-3}$

(q) $\displaystyle\lim_{x\to2^+}\frac{x}{x^2-4}$

(r) $\displaystyle\lim_{x\to2^-}\frac{x}{x^2-4}$

(s) $\displaystyle\lim_{y\to6^+}\frac{y+6}{y^2-36}$

(t) $\displaystyle\lim_{y\to6^-}\frac{y+6}{y^2-36}$

(u) $\displaystyle\lim_{x\to4^+}\frac{3-x}{x^2-2x-8}$

(v) $\displaystyle\lim_{x\to4^-}\frac{3-x}{x^2-2x-8}$

(x) $\displaystyle\lim_{x\to3^-}\frac{1}{|x-3|}$

(y) $\displaystyle\lim_{x\to3^+}\frac{1}{|x-3|}$

[R.a.$+\infty$;b.0;c.1/2;d.$-\infty$;e.$+\infty$;f.$+\infty$;g.$+\infty$;h.-1;i.-1/2;j.10/3;k.0;l.$-\sqrt{2}$; m. $\sqrt[3]{3/2}$;n.-1/2;o.$+\infty$;p.$-\infty$;
q.$+\infty$;r.$-\infty$;s.$+\infty$;t.$-\infty$;u.$-\infty$;v.$+\infty$; x.$+\infty$y.$+\infty$.]

(11) Calcule os limites a seguir.

(a) $\displaystyle\lim_{x\to 0} \frac{\operatorname{sen}(4x)}{3x}$

(b) $\displaystyle\lim_{x\to 3} \frac{\operatorname{sen}(10x)}{\operatorname{sen}(7x)}$

(c) $\displaystyle\lim_{x\to 0} \frac{\operatorname{tg} ax}{x}$

(d) $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{x}$

(e) $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{x^2}$

(f) $\displaystyle\lim_{x\to 0} \frac{\cos 2x - \cos 3x}{x^2}$

(g) $\displaystyle\lim_{t\to \infty} \left(\frac{2t+3}{2t+1}\right)^{n+1}$

(h) $\displaystyle\lim_{x\to \frac{3\pi}{2}} \frac{2x^2 - x}{3x}$

(i) $\displaystyle\lim_{x\to \infty} \left(1+\frac{10}{x}\right)^{x}$

(j) $\displaystyle\lim_{x\to 2} \frac{10^{x-2}-1}{x-2}$

(k) $\displaystyle\lim_{x\to 2} \frac{5^x - 25}{x-2}$

(l) $\displaystyle\lim_{x\to 0} \frac{e^{-ax}-e^{-bx}}{sen(ax)-sen(bx)}$

[R.4/3;10/7;a;0;1/2;5/2;e;e;e^{10};ln 10;25 ln 5;$b-a$;1]

Bom Estudo!