

**UNIVERSIDADE FEDERAL DE OURO PRETO**  
**INSTITUTO DE CIÊNCIAS EXATAS E BIOLÓGICAS**  
**DEPARTAMENTO DE MATEMÁTICA**

Sétima Lista de Exercícios de Cálculo Diferencial e Integral I - MTM122

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- (1) Escreva as formulas para a regra do produto, regra do quociente, regra da cadeia e de derivadas de funções inversas.
- (2) Para cada função  $f(x)$  à esquerda da tabela abaixo complete a tabela com a derivada  $Df(x)$  à direita.

$f(x)$	$Df(x)$
$c$	
$x^{-1}$	
$x^{1/2}$	
$x$	
$x^2$	$2x$
$x^3$	
$3x^4$	
$ax^n$	
$\text{sen}(x)$	$\text{cos}(x)$
$\text{cos}(x)$	
$\text{tg}(x)$	
$\text{sec}(x)$	
$\text{cossec}(x)$	
$\text{cotg}(x)$	
$e^x$	
$\ln(x)$	
$\text{senh}(x)$	
$\text{cosh}(x)$	
$\text{sen}^2(x)$	$2\text{sen}(x)\text{cos}(x)$
$\text{cos}^2(x)$	
$a^x$	
$\log_a(x)$	
$\text{arcsen}(x)$	
$\text{arccos}(x)$	
$\text{arctg}(x)$	
$\text{arcsec}(x)$	
$\text{arccossec}(x)$	
$\text{arccotg}(x)$	
$\text{argsenh}(x)$	
$\text{argcosh}(x)$	

(3) Derive e simplifique.

- (1)  $y = x^5$  (2)  $y = x$  (3)  $y = x^{10}$  (4)  $g(x) = 8x^2$   
 (5)  $g(z) = -2z^7$  (6)  $f(x) = 3x^4 + 8x + 5$  (7)  $g(y) = 9y^5 - 4y^2 + 2y + 7$  (8)  $y = (2x^3 - 1)(x^4 + x^2)$   
 (9)  $F(t) = \frac{1}{2}(t^2 + 5)(t^6 + 4t)$  (10)  $y = \frac{2x^4 - 3}{x^2 - 5x + 3}$  (11)  $y = 1/x$  (12)  $y = x^{-n}, n \in \mathbb{Z}_+$   
 (13)  $\frac{t-1}{t+1}$  (14)  $s(t) = s_0 + vt$  (15)  $\theta(t) = \theta_0 + \omega t$  (16)  $A(\ell) = \frac{\ell^2 \sqrt{3}}{2}$   
 (17)  $A_\circ(r) = \pi r^2$  (18)  $y = y(x) > 0$  e  $x^2 + y^2 = 1$  (19)  $y(x) = (x+a)^2$  (20)  $y = -1$   
 (21)  $y(x) = (x^2 + a)^2$  (22)  $(x^2 + 5x + 2)^7$  (23)  $y = \left(\frac{3x+2}{2x+1}\right)^5$  (24)  $(3x^2 + 1)^3(x - x^2)^2$   
 (25)  $y = 5\sqrt{x^2 + 3}$  (26)  $g(t) = \frac{t^2}{\sqrt[3]{t^3 + 1}}$  (27)  $y = x^8 + (2x+4)^3 + \sqrt{x}$  (28)  $z(\xi) = \frac{\xi + 1}{\sqrt{\xi^2 - 3}}$   
 (29)  $y = 3x(8x^3 - 2)$  (30)  $y = \sqrt[3]{6x^2 + 7x + 2}$  (31)  $y = e^x$  (32)  $y = a^x$   
 (33)  $y = \sinh(x)$  (34)  $y = \cosh(x)$  (35)  $y = \operatorname{sen}(x)$  (36)  $y = \cos(x)$   
 (37)  $y = \operatorname{tg}(x)$  (38)  $y = \operatorname{sec}(x)$  (39)  $y = \operatorname{cossec}(x)$  (40)  $y = \operatorname{cotg}(x)$   
 (41)  $y = \operatorname{arcsen}(x)$  (42)  $y = \operatorname{arccos}(x)$  (43)  $y = \operatorname{arctg}(x)$  (44)  $y = \ln(x)$   
 (45)  $y = \operatorname{argsenh}(x)$  (46)  $f(x) = 3x^2 + 8x + 1, f''(x) = ?$  (47)  $y = \operatorname{tg}(x), \dot{y} = ?$  (48)  $y = \operatorname{sen}(x), y^{vii}(x) = ?$   
 (49)  $Y(s) = e^{\frac{s}{2}}, Y^{(n)}(s) = ?$  (50)  $y = 3x^5 + 8x^2, f^{(n)}(x); n \in \mathbb{N}$

Respostas. 1.5  $3.10x$  5.  $-14x^6$  7.  $45y^4 - 8y^2 + 2$  9.  $t^2(t^5 + 4) + t(3t^4 + 2)(t^2 + 5)$  11.  $-1/x^2$  13.  $2/(t+1)^2$  15.  $\omega$  17.  $A'_\circ(r) = 2\pi r$  19.  $2(x+a)$   
 21.  $4x(x^2 + a)$  23.  $-5(3x+2)^4/(2x-1)^6$  25.  $5x/\sqrt{x^2+3}$  27.  $8x^7 + 6(2x+4)^2 + \sqrt{x}/(2x)$  29.  $96x^3 - 6$  41.  $1/\sqrt{1-x^2}$  43.  $1/(1+x^2)$  45.  $1/\sqrt{1+x^2}$   
 47.  $2 \sec^2(x) \operatorname{tg}(x)$  49.  $e^{s/2}/2^n$

(4) Derive e simplifique.

- (1)  $y = (x^4 - 3x^2 + 5)^3$       (2)  $y = \cos(\operatorname{tg}(x))$       (3)  $y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}}$       (4)  $y = \frac{3x-2}{\sqrt{2x+1}}$
- (5)  $y = 2x\sqrt{x^2+1}$       (6)  $y = \frac{e^x}{1+x^2}$       (7)  $y = e^{\operatorname{sen}(2\theta)}$       (8)  $y = e^{-t}(t^2 - 2t + 2)$
- (9)  $y = \frac{t}{1-t^2}$       (10)  $y(x) = e^{mx} \cos(nx)$       (11)  $y = \sqrt{x \cos \sqrt{x}}$       (12)  $y = (\operatorname{arcsen} 2x)^2$
- (13)  $y = \frac{e^{1/x}}{x^2}$       (14)  $y = \frac{1}{\operatorname{sen}(x - \operatorname{sen} x)}$       (15)  $y = x \ln x - x$       (16)  $y = \ln(\operatorname{cosec} 5x)$
- (17)  $y = \frac{\sec 2\theta}{1 + \operatorname{tg} 2\theta}$       (18)  $y = 2^x$       (19)  $y = e^{cx}(\operatorname{c} \operatorname{sen} x - \cos x)$       (20)  $y = \ln(x^2 e^x)$
- (21)  $y = 3^{x \ln x}$       (22)  $y = \sec(1 + x^2)$       (23)  $y = (1 - x^{-1})^{-1}$       (24)  $y = 1/\sqrt[3]{x + \sqrt{x}}$
- (25)  $y = 3^x$       (26)  $y = \sqrt{\operatorname{sen} \sqrt{x}}$       (27)  $y = \log_5(1 + 2x)$       (28)  $y = (\cos x)^x$
- (29)  $y = \ln \operatorname{sen} x - \frac{1}{2} \operatorname{sen}^2 x$       (30)  $y = \frac{(x^2 + 1)^4}{(2x + 1)^3(3x - 1)^5}$       (31)  $y = x \operatorname{arctg}(4x)$       (32)  $y = e^{\cos x} + \cos(e^x)$
- (33)  $y = \ln |\sec 5x + \operatorname{tg} 5x|$       (34)  $y = 10^{\operatorname{tg} \pi \theta}$       (35)  $y = \operatorname{cotg}(3x^2 + 5)$       (36)  $y = \sqrt{t \ln(t^4)}$
- (37)  $y = \operatorname{sen}(\operatorname{tg} \sqrt{1 + x^3})$       (38)  $y = \operatorname{arctg}(\operatorname{arcsen} \sqrt{x})$       (39)  $y = \operatorname{tg}^2(\operatorname{sen}(\theta))$       (40)  $y = x e^x$
- (41)  $y = \frac{\sqrt{x+1}(2-x)^5}{(x+3)^7}$       (42)  $y = \frac{(x+\lambda)^4}{x^4 + \lambda^4}$       (43)  $y = x \operatorname{senh}(x^2)$       (44)  $y = \frac{\operatorname{sen} mx}{x}$
- (45)  $y = \ln(\operatorname{cosh} 3x)$       (46)  $y = \ln \left| \frac{x^2 + 4}{2x + 5} \right|$       (47)  $y = \operatorname{argcosh}(\operatorname{senh} x)$       (48)  $y = x \operatorname{arctg} \sqrt{x}$
- (49)  $y = \cos(e^{\sqrt{\operatorname{tg}(3x)}})$       (50)  $y = \operatorname{sen}^2(\cos \sqrt{\operatorname{sen} \pi x})$

Respostas. 1.  $6x(x^4 - 3x^2 + 5)^2(2x^2 - 3)$  3.  $1/(2\sqrt{x}) - 4/(3\sqrt[3]{x^7})$  5.  $2(2x^2 + 1)/\sqrt{x^2 + 1}$  7.  $2 \cos 2\theta e^{\operatorname{sen} 2\theta}$  9.  $(t^2 + 1)/(1 - t^2)^2$  11.  $(\cos \sqrt{x} - \sqrt{x} \operatorname{sen} \sqrt{x})/2\sqrt{x}$  13.  $[e^{1/x}(1 + 2x)]/x^4$  15.  $\ln x$  17.  $2 \sec 2\theta(\operatorname{tg} 2\theta - 1)/(1 + \operatorname{tg} 2\theta)^2$  19.  $(1 + c^2)e^{cx} \operatorname{sen} x$  21.  $3^x \ln x (\ln 3)(1 + \ln x)$  23.  $-(x-1)^{-2}$  25.  $3^x \ln 3$  27.  $2/(1+2x) \ln 5$  29.  $\operatorname{cotg} x - \operatorname{sen} x \cos x$  31.  $4x/(1+16x^2) + \operatorname{arctg}(4x)$  33.  $5 \sec 5x$  35.  $-6x \operatorname{cosec}^2(3x^2 + 5)$  37.  $3x^2 \cos(\operatorname{tg} \sqrt{1 + x^3})(\sec^2 \sqrt{1 + x^3})/2\sqrt{1 + x^3}$  39.  $2 \cos \theta \operatorname{tg}(\operatorname{sen} \theta) \sec^2(\operatorname{sen} \theta)$  41.  $(x-2)^4(3x^2 - 55x - 52)/2\sqrt{x+1}(x+3)^8$  43.  $2x^2 \cosh(x^2) + \operatorname{senh}(x^2)$  45.  $3 \operatorname{tgh} 3x$  47.  $\operatorname{cosh} x / \sqrt{\operatorname{senh}^2 x - 1}$  49.  $(-3 \operatorname{sen}(e^{\sqrt{\operatorname{tg} 3x}}) e^{\sqrt{\operatorname{tg} 3x}} \sec^2(3x))/2\sqrt{\operatorname{tg} 3x}$

(5) Derive e simplifique.

- (1)  $y = (\cosh^2 x - \sinh^2 x)^{100}$  (2)  $f(x) = \arcsen(2\text{sen}(x) \cos(x))$  (3)  $f(x) = 2x^3 - 7x + 2$   
(4)  $k(x) = 1/(x^4 - x^2 + 1)$  (5)  $g(t) = \sqrt{6t + 5}$  (6)  $h(t) = 1/\sqrt{6t + 5}$   
(7)  $F(z) = \sqrt[3]{7z^2 - 4z + 3}$  (8)  $f(\omega) = \sqrt[5]{3\omega^2}$  (9)  $G(x) = 6/(3x^2 - 1)^4$   
(10)  $H(x) = (3x^2 - 1)^4/6$  (11)  $F(y) = (y^2 - y^{-2})^{-2}$  (12)  $h(z) = [(z^2 - 1)^5 - 1]^5$   
(13)  $g(x) = \sqrt[5]{(3x + 2)^4}$  (14)  $P(x) = (x + x^{-1})^2$  (15)  $r(s) = \left(\frac{8s^2 - 4}{1 - 9s^3}\right)$   
(16)  $g(w) = \frac{(w - 1)(w - 3)}{(w + 1)(w + 3)}$  (17)  $F(x) = (x^6 + 1)^5(3x + 2)^3$  (18)  $k(z) = (z^2 + (z^2 + 9)^{1/2})^{1/2}$   
(19)  $g(y) = \sqrt{1 + \cos 2y}$  (20)  $p(x) = (2x^4 + 3x^2 - 1)/x^2$  (21)  $f(x) = \text{sen}^2(4x^3)$   
(22)  $H(t) = (1 + \text{sen} 3t)^3$  (23)  $h(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$  (24)  $K(r) = \sqrt{r}\sqrt{r + 1}\sqrt{r + 2}$   
(25)  $f(x) = \sqrt[3]{2x + 3}/\sqrt{3x + 2}$  (26)  $f(x) = 6x^2 - (5/x) + (2/\sqrt[3]{x^2})$  (27)  $g(z) = (9z^{5/3} - 5z^{3/5})^3$   
(28)  $F(t) = (5t^2 - 7)/(t^2 + 2)$  (29)  $k(s) = (2s^2 - 3s + 1)(9s - 1)^4$  (30)  $H(x) = |\cos x|$   
(31)  $f(w) = \sqrt{(2w + 5)/(7w - 9)}$  (32)  $S(t) = \sqrt{t^2 + t + 1}\sqrt[3]{4t - 9}$  (33)  $P(\theta) = \theta^2 \cos^2 \theta^2$   
(34)  $g(v) = 1/(1 + \cos^2 2v)$  (35)  $g(x) = (\cos \sqrt[3]{x} - \text{sen} \sqrt[3]{x})^3$  (36)  $f(x) = \text{sen}(\cos 5x)$   
(37)  $y = 5x^3 + 4\sqrt{x}$  (38)  $y = 2x^2 - 3x - \cos 5x$  (39)  $y = x^3 - x^2 - 5x + 2$   
(40)  $y = 3x^2 - 2x - 5$  (41)  $f(x) = \text{arctg}(\theta x) - k$  (42)  $G(x) = 3x^{-2} - 2x - \sec \theta$

Respostas. 1.0. 2.2.  $3.6x^2 - 7.5.3/\sqrt{6t + 5}$  7.(1/3)(7z^2 - 4z + 3)^{-2/3}(14z - 4) 9. -144x/(3x^2 - 1)^5 11. -2(y^2 - y^{-2})^{-3}(2y - 2y^{-3}) 13.(12/5)(3x + 2)^{-1/5} 15.4(8s^2 - 4)^3(72s^4 - 108s^2 + 16s)/(1 - 9s^3)^5 17.(x^6 + 1)^4(3x + 2)^2(99x^6 + 60x^5 + 9) 19. -sen2y/\sqrt{1 + \cos 2y} 21.24x^2sen(4x^3)cos(4x^3) = 12x^2sen8x^3 23.(1/2)\sqrt{x + \sqrt{x + \sqrt{x}}}(1 + (2\sqrt{x} + 1)/(4\sqrt{x}\sqrt{x + \sqrt{x}})) 25.[(2/3)(3x + 2)^{1/2}(2x + 3)^{-2/3} - (3/2)(2x + 3)^{1/3}(3x + 2)^{-1/2}]/[3x + 2] 27.3(9z^{5/3} - 5z^{3/5})^2(15z^{2/3} - 3z^{-2/5}) 29.(9s - 1)^3(108s^2 - 139s + 39) 31. -53/2\sqrt{(2w + 5)(7w - 9)^3} 33.2\theta \cos \theta^2[\cos \theta^2 - 2\theta^2 \text{sen} \theta^2] 35. -x^{-2/3}(\cos \sqrt[3]{x} + \text{sen}[3x])(\cos[3x] - \text{sen}[3x])^2 37.15x^2 + 2/\sqrt{x} 39.3x^2 - 2x - 5 41.\theta/(1 + \theta^2 x^2)

Bom Estudo!