

Universidade Federal de Viçosa

Centro de Ciências Exatas

Departamento de Matemática

Gabarito 1ª Lista - MAT 137 - Introdução à Álgebra Linear 2013/I

1. a) Não, b) Não, c) Sim, ordem 4×5 , d) Não

2. a) $A_{5 \times 6}$, b) $B_{3 \times 6}$, c) $C_{3 \times 4}$, d) $D_{4 \times 3}$, e) $E_{3 \times 5}$

3. a) $A_{4 \times 5}$, b) $a_{23} = 11$, $a_{35} = 3$, $a_{43} = -4$.

4. $A_{3 \times 2}$, $B_{2 \times 3}$, $C_{3 \times 2}$, $D_{2 \times 3}$, $E_{3 \times 2}$

5. $c_{32} = 18$, $d_{43} = 23$.

$$6. A = \begin{bmatrix} 3 & -4 & -7 & -10 \\ -2 & 8 & -5 & -8 \\ -5 & -1 & 15 & -6 \\ -8 & -4 & 0 & 24 \end{bmatrix}$$

7. a) $A^2 = I$, b) $A^3 = A$, c) $A^{31} = A$, d) $A^{42} = I$

8. $x = -1$, $y = 1$

9. a) $x = 4$, b) $x = 12$, $y = -8$, $z = -4$, c) $x = 2$, $y = -7$, $z = -2$
 $x = -2$, $y = -3$, $z = 10$

$$10. a) \begin{bmatrix} 22 & -6 & 8 \\ -2 & 4 & 6 \\ 10 & 0 & 4 \end{bmatrix}, b) \begin{bmatrix} 7 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix}, c) A = \begin{bmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -6 \end{bmatrix}, d) \begin{bmatrix} 10 & -6 \\ -14 & 2 \\ -1 & -8 \end{bmatrix}$$

$$11. a) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

$$b) 4 \text{ matrizes: } \begin{bmatrix} \sqrt{5} & 0 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} -\sqrt{5} & 0 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} \sqrt{5} & 0 \\ 0 & -3 \end{bmatrix}, \begin{bmatrix} -\sqrt{5} & 0 \\ 0 & -3 \end{bmatrix}.$$

$$c) \text{ Não, considere } A = \begin{bmatrix} -2 & 0 \\ 1 & 1 \end{bmatrix}.$$

12.

13.

$$14. a) \pm 1, \quad b) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{ e } \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

15. $m = \pm 1$

16. Ortogonais: A , C e D .

Não ortogonais: B .

17.

18.

19.

20.

21. A matrix A também é diagonal.

$$22. A^{-1} = \begin{bmatrix} a_{11}^{-1} & 0 & \cdots & 0 \\ 0 & a_{12}^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn}^{-1} \end{bmatrix}$$

23.

24. Sim

25.

26. a)F, b)V, c)V, d)V, e)F, f)V, g)V, h)F, i)V

27. a) 4^5 , b) P é inversível, c) -9 , d) Q é inversível

28. -5

29. a)576,

$$b)A^{-1} = \begin{bmatrix} 1 & \frac{5}{2} & \frac{17}{8} & -\frac{31}{12} \\ 0 & \frac{1}{2} & \frac{3}{8} & -\frac{5}{12} \\ 0 & 0 & \frac{1}{4} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{3} \end{bmatrix},$$
$$c)B^{-1} = \begin{bmatrix} -\frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{4} & -\frac{1}{4} & 0 & 0 \\ -\frac{6}{7} & -\frac{1}{1} & -1 & 0 \\ \frac{6}{25} & \frac{3}{3} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{6}{24} & -\frac{8}{8} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix},$$

$$d)(AB)^{-1} = \begin{bmatrix} -\frac{215}{288} & -\frac{37}{288} & -\frac{67}{144} & -\frac{25}{72} \\ -\frac{23}{32} & -\frac{5}{32} & -\frac{3}{16} & -\frac{1}{8} \\ -\frac{5}{6} & -\frac{1}{6} & -\frac{1}{3} & -\frac{1}{6} \\ \frac{31}{24} & \frac{5}{24} & -\frac{1}{12} & -\frac{1}{6} \end{bmatrix},$$

$$e)\det C = 0 \text{ ou } \det C = \frac{1}{16}.$$

$$30. \det Q = (-2)^n$$

$$31. \text{ a) } 58, \quad \text{ b) } 58, \quad \text{ c) } 3364 \quad \text{ d) } A^{-1} = \begin{bmatrix} \frac{25}{29} & -\frac{32}{29} & -\frac{13}{29} & \frac{10}{29} \\ \frac{7}{29} & -\frac{2}{29} & \frac{1}{29} & -\frac{3}{29} \\ -\frac{157}{58} & \frac{165}{58} & \frac{77}{58} & -\frac{14}{29} \\ -\frac{73}{58} & \frac{83}{58} & \frac{31}{58} & -\frac{3}{58} \end{bmatrix},$$

$$e) 58, \quad f) 3. \quad \begin{bmatrix} 36 & 23 & 35 & 32 \\ 23 & 25 & -2 & 17 \\ 35 & -2 & 95 & 47 \\ 32 & 17 & 47 & 50 \end{bmatrix}$$

$$32. p(x) = x^3 - 2x^2 - x + 3 \text{ e } A^{-1} = -\frac{1}{3}(A^2 - 2A - I).$$

$$33. \text{ a) } -123, \quad \text{ b) } 1 + a + b + c, \quad \text{ c) } -c^4 + c^3 - 16c^2 + 8c - 2, \quad \text{ d) } -5 \quad \text{ e) } -120, \quad \text{ f) } -120$$

$$34. \text{ a) } x = 0, -1, 1/2, \quad \text{ b) } x = 40/11, \quad \text{ c) } x = \frac{3}{4} \pm \frac{1}{4}\sqrt{33}.$$

$$35. \det(A) = a_{41}a_{32}a_{23}a_{14}.$$

36.

$$37. \text{ a) } A^{-1} = \begin{bmatrix} \frac{29}{152} & \frac{11}{152} & -\frac{1}{8} \\ -\frac{21}{152} & \frac{13}{152} & \frac{1}{8} \\ \frac{27}{152} & \frac{5}{132} & \frac{1}{8} \end{bmatrix},$$

$$\text{b) } A^{-1} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\text{c) } A^{-1} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & -\frac{5}{2} & 1 & -\frac{1}{2} \\ -1 & 2 & -1 & 0 \end{bmatrix},$$

$$\text{d) } A^{-1} = \begin{bmatrix} 0 & 0 & \frac{1}{4} & 0 \\ 0 & -1 & \frac{1}{2} & 0 \\ \frac{1}{6} & \frac{5}{6} & -\frac{2}{13} & 0 \\ \frac{6}{2} & \frac{6}{16} & -\frac{24}{53} & \frac{1}{3} \\ \frac{9}{9} & \frac{9}{9} & -\frac{36}{36} & \frac{3}{3} \end{bmatrix},$$

38.

$$\text{39. a) } A^{-1} = \begin{bmatrix} -\frac{1}{8} & \frac{3}{8} & -\frac{1}{8} \\ -\frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{4} & 0 \end{bmatrix},$$

$$\text{b) } A^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{1}{14} \\ -\frac{1}{7} & \frac{3}{14} \end{bmatrix},$$

$$\text{c) } A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix},$$

$$\text{d) } A^{-1} = \frac{1}{51} \begin{bmatrix} -5 & -28 & -6 \\ -2 & 16 & 1 \\ -2 & -1 & 1 \end{bmatrix}$$

$$40. \det(B) = 1 \text{ e } B^{-1} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

41. a)F, b)F, c)V, d)F, e)F, f)F, g)V, h)V, i)V.

42. Para o item (a) faça o produto matricial AB , onde $A = \begin{bmatrix} 249 & 12 & 52 & 52 \end{bmatrix}$ e $B = \begin{bmatrix} 400 & 600 & 450 & 650 \\ 350 & 550 & 500 & 600 \\ 350 & 600 & 500 & 650 \\ 450 & 500 & 400 & 700 \end{bmatrix}$. No item (b) basta somar 60% em cada entrada da matriz resultante.

$$43. \text{a) } A = \begin{bmatrix} 20 & 15 & 30 \end{bmatrix} \cdot \begin{bmatrix} 50 & 15 & 6 & 70 & 25 \\ 500 & 1 & 5 & 20 & 30 \\ 200 & 8 & 7 & 50 & 40 \end{bmatrix}$$

b) Os elementos de AB representam o valor total de compra e o preço total de transporte de todos os materiais utilizados na construção de todos os estabelecimentos.

$$44. \text{Faça os produtos } AB \text{ e } AC, \text{ onde } A = \begin{bmatrix} 6 & 7 & 5 & 8 \end{bmatrix}, B = \begin{bmatrix} 25 & 15 & 70 \\ 30 & 25 & 40 \\ 60 & 10 & 55 \\ 15 & 30 & 60 \end{bmatrix} \text{ e}$$

$$C = \begin{bmatrix} 7,5 \\ 5 \\ 4,5 \\ 6,5 \end{bmatrix}.$$

45. Cada linha representa o custo total de cada produto e as colunas representam esses custos totais em cada cidade.

$$46. A \cdot B^T = \begin{bmatrix} 71000 & 47000 & 110000 \\ 97000 & 29000 & 147000 \\ 114000 & 655000 & 176000 \end{bmatrix}$$

47. a)F, b)F, c)F, d)V, e)F, f)F, g)V, h)F, i)V, j)V, k)V, l)F.