Gabarito da Segunda Lista de Álgebra Elementar

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1. Módulo, argumento principal, a forma polar e a representação gráfica:

   (a) $|z| = 1$, $\theta = \frac{\pi}{3}$, $z = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

   (b) $|z| = 1$, $\theta = \frac{\pi}{6}$, $z = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

   (c) $|z| = 1$, $\theta = \frac{7\pi}{4}$, $z = \cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}$

   (d) $|z| = 7$, $\theta = 0$, $z = 7(\cos 0 + i\sin 0)$

   (e) $|z| = 2$, $\theta = \frac{3\pi}{2}$, $z = 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

   (f) $|z| = 3$, $\theta = \pi$, $z = 3(\cos\pi + i\sin\pi)$

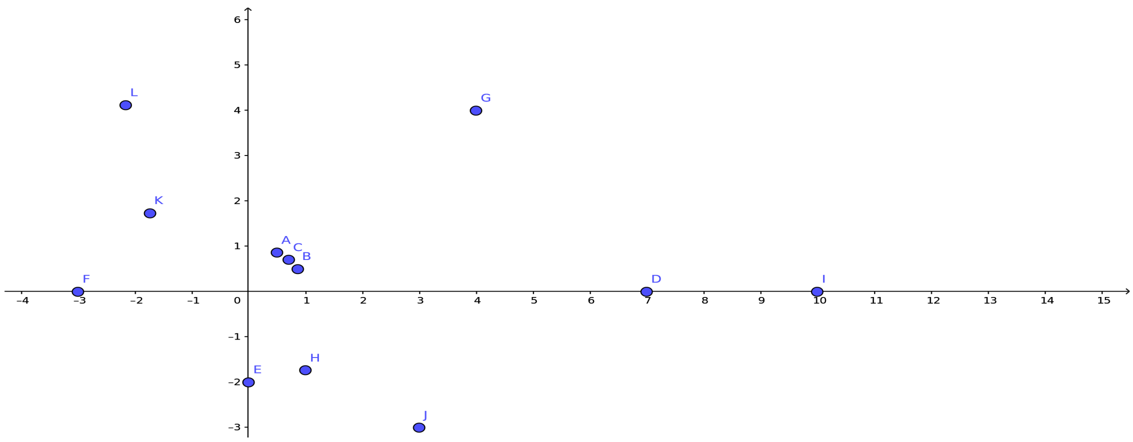
   (g) $|z| = 4\sqrt{2}$, $\theta = \frac{\pi}{4}$, $z = 4\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

   (h) $|z| = 2$, $\theta = \frac{5\pi}{3}$, $z = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

   (i) $|z| = 10$, $\theta = \frac{\pi}{2}$, $z = 10\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

   (j) $|z| = 3\sqrt{2}$, $\theta = \frac{7\pi}{4}$, $z = 3\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

   (k) $|z| = \sqrt{6}$, $\theta = \frac{3\pi}{4}$, $z = \sqrt{6}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$



2. Módulo de z:

(a) $|z| = \sqrt{13}$

(b) $|z| = \sqrt{5}$

(c) $|z| = \dfrac{3}{2}$

(d) $|z| = 1$

(e) $|z| = 1$

3. Forma algébrica $z = a + bi$, em que $a, b \in \mathbb{R}$.

(a) $z = 2$

(b) $z = 5i$

(c) $z = -\sqrt{3} + i$

(d) $z = i$

4. Módulo dos números abaixo:

(a) $|z| = |2 - i| \cdot |3 - 3i| = \sqrt{5} \cdot \sqrt{18} = 3\sqrt{10}$

(b) $|z| = |-1 + i\sqrt{3}|^7 = \left(\sqrt{4}\right)^7 = 2^7 = 128$

(c) $|z| = \dfrac{|2 + 3i|}{|2 - i|} = \dfrac{\sqrt{13}}{\sqrt{5}} = \sqrt{\dfrac{13}{5}}$

5. Forma trigonométrica:

(a) $z = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^2 = \cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

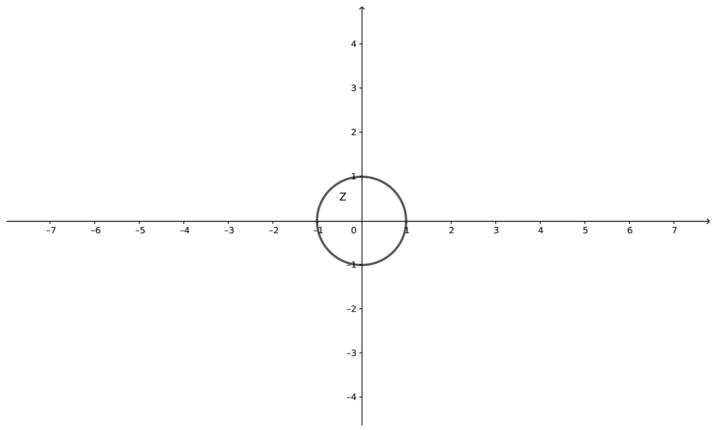
(b) $z = \dfrac{i}{1+i} = \dfrac{i\,(1-i)}{(1+i)\,(1-i)} = \dfrac{1+i}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

(c) $z = 3(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6})$

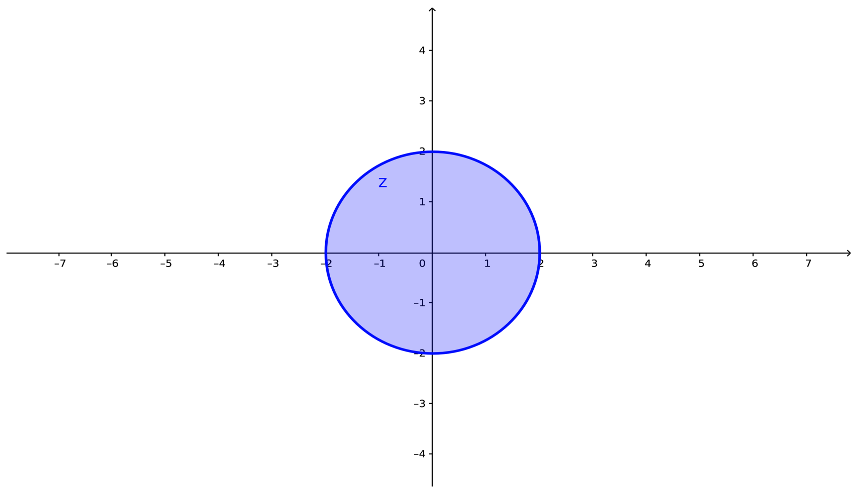
(d) $z = 2(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6})$

6. Represente geometricamente no plano de Argand-Gauss os seguintes subconjuntos de $\mathbb{C}$:

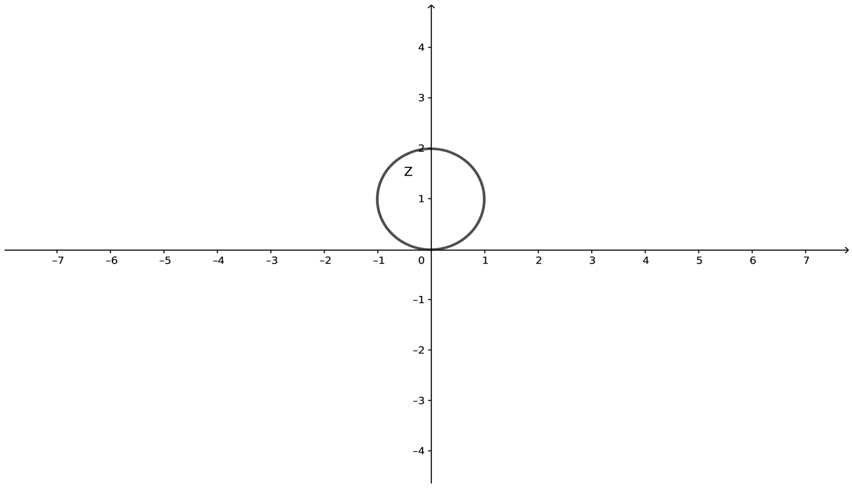
(a) $z = \{z \in \mathbb{C} \mid |z| = 1\}$



(b) $z = \{z \in \mathbb{C} \mid |z| \leqslant 2\}$



(c) $z = \{z \in \mathbb{C} \mid |z - i| = 1\}$



7. Represente no plano de Argand-Gauss o conjunto de todos os números complexos da forma $z = \rho(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$ em que $\rho \in \mathbb{R}$.