Gabarito da Terceira Lista de Álgebra Elementar
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1. Resultados na forma algébrica $z = a+bi$ e na forma trigonométrica $z = \rho\,(\cos\theta + i\sin\theta)$:

   (a) Forma algébrica $z = -2i$
   Forma trigonométrica $z = 2 \cdot \left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

   (b) Forma algébrica $z = -8i$
   Forma trigonométrica $z = 8 \cdot \left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

   (c) Forma algébrica $z = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$
   Forma trigonométrica $z = 1 \cdot \left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

2. Calcule as potências a seguir, deixando o resultado na forma algébrica $z = a + bi$:

   (a) $z = -1$

   (b) $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

   (c) $z = -1000$

   (d) $z = -8i$

   (e) $z = \frac{1}{512}i$

   (f) $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

   (g) $z = \frac{1}{8}i$

   (h) $z = \frac{1}{256} - \frac{1}{256}i$

   (i) $z = 512 + 512i$

   (j) $z = -2^{99} - 2^{99}\sqrt{3}i$

   (k) $z = -512 - 512\sqrt{3}i$

   (l) $z = 6^{100}\sqrt{3} + 6^{100}\sqrt{3}i$

   (m) $z = 1 - 2^{100} + 2^{100}\sqrt{3}i$

   (n) $z = -\frac{\sqrt{3}}{256} - \frac{1}{256}i$

3. Menor número natural n para o qual $\left(\sqrt{3} - i\right)^n$ é:

   (a) real e positivo é o número $n = 12$;

   (b) real e negativo é o número $n = 6$;

   (c) imaginário puro é o número $n = 3$.

4. $z_1 = -2\sqrt{2}$ e $z_2 = 2\sqrt{2}$ são as raízes quadradas de 8;
   $z_1 = -1 + \sqrt{3}i$, $z_2 = -1 - \sqrt{3}i$ e $z_3 = 2$ são as raízes cúbicas de 8;
   $z_1 = \sqrt[4]{2}i$, $z_2 = -\sqrt[4]{2}$, $z_3 = -\sqrt[4]{2}i$ e $z_4 = \sqrt[4]{2}$ são as raízes quartas de 8.

5. Raízes:

   (a) $z_1 = 3 + 4i$ e $z_2 = -3 - 4i$

   (b) $z_1 = 3 + 2i$ e $z_2 = -3 - 2i$

   (c) $z_1 = \sqrt[6]{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$, $z_2 = \sqrt[6]{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ e $z_3 = \sqrt[6]{2}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$

   (d) $z_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$, $z_2 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$, $z_3 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ e $z_4 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

   (e) $z_1 = \sqrt[6]{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$, $z_2 = \sqrt[6]{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ e $z_3 = \sqrt[6]{2}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$

   (f) $z_1 = -2\sqrt{2} + 2\sqrt{2}i$ e $z_2 = 2\sqrt{2} - 2\sqrt{2}i$

   (g) $z_1 = 3$, $z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$, $z_3 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$, $z_4 = -3i$, $z_5 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ e $z_6 = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

   (h) $z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ e $z_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

   (i) $z_1 = -\frac{\sqrt[3]{16}}{4}i$, $z_2 = -\frac{\sqrt[3]{16}\sqrt{3}}{8} + \frac{\sqrt[3]{16}}{8}i$ e $z_3 = \frac{\sqrt[3]{16}\sqrt{3}}{8} + \frac{\sqrt[3]{16}}{8}i$

6. Valores de x:

   (a) $x = 1$ ou $x = -1$

   (b) $x = i$ ou $x = -i$

   (c) $x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ ou $x = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

   (d) $x = \frac{\sqrt{3}}{2} + \frac{1}{2}i$, $x = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$ ou $x = -i$

   (e) $x = 3$, $x = -\frac{3}{2} + \frac{3\sqrt{3}}{2}i$ ou $x = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$

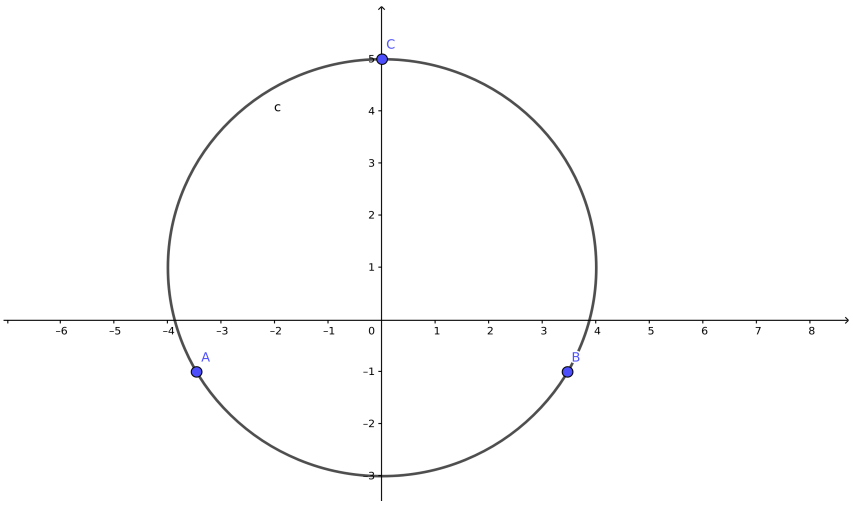
   (f) $x = \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$, $x = -\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$, $x = -\sqrt{2}$, $x = -\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i$, $x = \frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i$ ou $x = \sqrt{2}$

   (g) $x = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$, $x = \sqrt{2}i$, $x = -\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}i$, $x = -\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$, $x = -\sqrt{2}i$ ou $x = \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}i$

7. $z_1 = 2 + 2\sqrt{3}i$, $z_2 = -2\sqrt{3} + 2i$, $z_3 = -2 - 2\sqrt{3}i$ e $z_4 = 2\sqrt{3} - 2i$.

8. As outras cinco raízes distintas são iguais a $1 + \sqrt{3}i$, $-1 + \sqrt{3}i$, $-1 - \sqrt{3}i$, $1 - \sqrt{3}i$ e 2.

9. **Observação** Mude o enunciado para: Represente graficamente os números $i + \sqrt[3]{-64i}$



Os números representados no gráfico são $A = -2\sqrt{3} - i$, $B = 2\sqrt{3} - i$ e $C = 4i$.

10. Os números complexos que estão nos outros vértices são iguais a $A = -\frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i$, $B = -5$, $C = -\frac{5\sqrt{2}}{2} - \frac{5\sqrt{2}}{2}i$, $D = -5i$, $E = \frac{5\sqrt{2}}{2} - \frac{5\sqrt{2}}{2}i$, $F = 5$ e $G = \frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i$.